Solution

Class 10 - Mathematics

Mathematics Class X - 2020-21, Paper 1

Part A

1. Let α and β be the two zeros of required quadratic polynomial. Then, we have,
   $\alpha + \beta = 3, \alpha\beta = 2$
   Therefore, required quadratic polynomial is,
   $x^2 - (\alpha + \beta)x + \alpha\beta$
   i.e $x^2 - 3x + 2$
2. The given quadratic polynomial is x^2 - 3
   Here, a = 1, b = 0, c = -3
   Product of zeroes = $\frac{c}{a}$ = -3
3. The number is $\frac{64}{455}$
   Factorize the denominator we get,
   455 = 5 $\times$ 7 $\times$ 13
   Since, the denominator is not in the form of $2^m \times 5^n$, and it also contains 7 and 13 as its factors,
   Its decimal expansion will be non-terminating repeating.
   OR
   The HCF of the two no. is 50.
   Step-by-step explanation:
   Let the LCM be x and HCF be y
   Therefore, by given condition x = 9y ...(1)
   and x + y = 500
   Using (1) in (2), we get
   9y + y = 500
   $\Rightarrow$ y = 50
4. Let a number of humans be x and deer be y.
   then, $x + y = 39$ ....(i)
   and $2x + 4y = 132$
   $x + 2y = 66$ .....(ii)
   On solving (i) and (ii), we get
   $-y = -27$
   $\Rightarrow$ y = 27, x = 12
5. For infinitely many solutions,
   $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{a}{-12} = \frac{-3}{a} = \frac{1}{2}$
   $\Rightarrow \frac{a}{-12} = \frac{-3}{a}$ and $\frac{-3}{a} = \frac{1}{2}$
   $\Rightarrow a^2 = 36 \; and \; a = -6$
   $\Rightarrow a = \pm 6 \; and \; a = -6$
   For $a = -6$, pair of given linear equations has infinitely many solutions.

6.

$$
\begin{array}{r|l}
 & 5y^3 - 2y^2 + \frac{5}{3}y \\
\hline
3y - 2 & 15y^4 - 16y^3 + 9y^2 - \frac{10}{3}y \\
 & \underline{\underset{-\quad +}{15y^4 - 10y^3}} \\
 & -6y^3 + 9y^2 - \frac{10}{3}y \\
 & \underline{\underset{+\quad -}{-6y^3 + 4y^2}} \\
 & 5y^2 - \frac{10}{3}y \\
 & \underline{\underset{-\quad +}{5y^2 - \frac{10}{3}y}} \\
 & \times
\end{array}
$$

So on dividing $15y^4 - 16y^3 + 9y^2 - \frac{10}{3}y$ by 3y - 2. Quotient = $5y^3 - 2y^2 + \frac{5}{3}y$ and remainder = 0

OR

It is given that One zero = - 8
and Sum of zeroes = 0
Since sum of zeroes $= \alpha + \beta$
$\therefore$ Other zero = 0 - (-8) = 8
Product of zeroes = $8 \times (-8)$ = - 64
Hence, Polynomial p(x) = x^2 - (S)x + P
= x^2 - 64

7. $\sqrt{21} = \sqrt{3} \times \sqrt{7}$ is an irrational number because $\sqrt{3}$ and $\sqrt{7}$ are irrational being square roots of prime numbers.

8. $3x^2 - 2x + 8 = 0$
Comparing it with $ax^2 + bx + c = 0$, we get
a = 3, b = -2 and c = 8
$\therefore D = b^2 - 4ac$
= $[(-2)^2 - 4(3)(8)]$
= (4 - 96)
= -92

9. $15x^2 - 28 = x \Rightarrow 15x^2 - x - 28 = 0$
$\Rightarrow 15x^2 - 21x + 20x - 28 = 0$
$\Rightarrow 3x(5x - 7) + 4(5x - 7) = 0$
$\Rightarrow$ (5x - 7) (3x + 4) = 0
$\Rightarrow$ 5x - 7 = 0 or 3x + 4 = 0
$\Rightarrow$ x = $\frac{7}{5}$ or x = $\frac{-4}{3}$

10. The given equations are 3x + 2y = 5 and 2x - 3y = 7.
Let $a_1 = 3$, $b_1 = 2$, $c_1 = 5$, $a_2 = 2$, $b_2 = -3$, $c_2 = 7$
$\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = -\frac{2}{3}$
$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations.

OR

Since, $a_1 = 1,\ b_1 = 2,\ c_1 = -4$
$a_2 = 2,\ b_2 = 1,\ c_2 = -5$
$\therefore \frac{a_1}{a_2} = \frac{1}{2}$ and $\frac{b_1}{b_2} = \frac{2}{1}$

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\therefore$ System of equations has unique solution.

11. The prime factorization of 4620 is

$$4620 = 2 \times 2 \times 3 \times 5 \times 7 \times 11$$
$$= 2^2 \times 3 \times 5 \times 7 \times 11$$

12. Smallest prime number = 2

Smallest composite number = 4

We know that

HCF $\times$LCM =The product of two numbers

= 2 $\times$ 4 = 8

13. We know that D = b^2 - 4 $\times$ a $\times$ c

$= (4\sqrt{3})^2 - 4 \times 4 \times 3$

= 16 $\times$ 3 - 48

= 48 - 48

=0

So, given quadratic equation has equal real roots.

OR

If Discriminant of quadratic equation is equal to zero, or more than zero then roots are real.

$3x^2$ -k$\sqrt{3}$x + 4 = 0

Compare with ax^2 + bx + c = 0

then a = 3, -k$\sqrt{3}$ and c = 4

D = b^2- 4ac

For real roots, b^2- 4ac > 0

$(-k\sqrt{3})^2 - 4 \times 3 \times 4 \geq 0$

$3k^2$- 48 $\geq$0

k^2 - 16 $\geq$0

(k - 4)(k + 4) $\geq$ 0

$\therefore$ k $\leq$ - 4 and k$\geq$ 4

14. We observe that the graph y = f (x) is a parabola opening upwards.

Therefore, f(x) is a quadratic polynomial in which coefficient of x^2 is positive.

15. No

The Condition for no solution is : $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (parallel lines)

Given pair of equations,

3x + y - 3 = 0

and $2x + \frac{2}{3}y = 2$

Comparing with ax + by + c = 0;

Here, a_1 = 3, b_1 = 1, c_1 = - 3;

And a_2 = 2, b_2 = 2/3, c_2 = - 2;

a_1 /a_2 = 2/6 = 3/2

b_1 /b_2 = 4/12 = 3/2

c_1 /c_2 = - 3/-2 = 3/2

Here, $a_1/a_2 = b_1/b_2 = c_1/c_2$, i.e coincident lines

Hence, the given pair of linear equations is coincident and having infinitely many solutions.

OR

Clearly, $(2x^2 - 3\sqrt{2}x + 6)$ is a quadratic polynomial.

$\therefore$ $(2x^2 - 3\sqrt{2}x + 6)$ is a quadratic equation.

16. Let , the number be x.

Then square of number will be x^2

According to question,

$x + 2x^2 = 21$

$\Rightarrow 2x^2 + x - 21 = 0$

Section I

17. i. (b) irrational
    ii. (a) terminating
    iii. (d) 2 and 5
    iv. (c) $(3+\sqrt{7})$ $(3-\sqrt{7})$
    v. (b) always an irrational number
18. i. (b) -10
    ii. (c) 1
    iii. (c) -4, 3
    iv. (d) $x^2 - 16$
    v. (c) $p(x) = x^2$
19. i. (a) x + 4y = 27; x+ 2y = 21
    ii. (a) Intersecting at exactly one point
    iii. (b) x = 15 ,Y = 3
    iv. (c) Bubbly paid additional ₹ 6 and Shristi paid ₹ 12.
    v. (b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
20. i. (a) 60 - 2x
    ii. (d) $60x - 2x^2$
    iii. (c) $x^2 - 30x + 125 = 0$
    iv. (b) the width = 15m
    v. (a) 10 or 20

Section II

21. The given equations may be written as
    $a^2x - b^2y = a^2b + ab^2$ .... (i)
    $ax - by = 2ab$ .... (ii)
    Multiplying (ii) by b and subtracting the result from (i),
    $(a^2 - ab)x = a^2b - ab^2$
    $\Rightarrow (a^2 - ab)x = b(a^2 - ab)$
    $\Rightarrow x = b.$
    Putting $x = b$ in (ii), we get
    $ab - by = 2ab$
    $\Rightarrow by = -ab$
    $\Rightarrow y = \frac{-ab}{b} = -a$
    Hence, x = b and y = -a.
22. The given system of linear equations is
    x - 2y = 8..................(1)
    5x - 10y = 10............(2)
    Here, a_1= 1, b_1 = -2, c_1 = -8;
    a_2 = 5, b_2 = -10, c_2 = -10
    We see that, $\frac{a_1}{a_2} = \frac{b_1}{b_2}$
    Hence, the given system of linear equations does not have a unique solution.
23. The given equation is
    $2x^2 - 3x + 5 = 0$
    Here, a = 2, b = -3, c = 5
    Therefore, discriminant = $b^2 - 4ac$
    $= (-3)^2 - 4(2)(5)$
    $= 9 - 40 = -31 < 0$
    So, the given equation has no real roots.

OR

$2x^2 - ax - a^2 = 0$

$\implies 2x^2 - 2ax + ax - a^2 = 0$

$\implies 2x(x-a) + a(x-a) = 0$

$\implies$ (2x+a)(x-a)=0

$\implies$ either 2x+a=0 or x-a=0

$\implies x = -\frac{a}{2}\, or\, x = a$

$\implies x = -\frac{a}{2}, a$

Thus, $x = -\frac{a}{2}$ and $x = a$ are two roots of the given quadratic equation.

24. Given numbers are: 17, 23 and 29

Since the three numbers are prime, we have

17 = 1 × 17

23 = 1 × 23

29 = 1 × 29

⇒ HCF = 1

and LCM = 17 × 23 × 29 = 11339

25. The given rational number is 3/8

It's seen that, 8 = $2^3 \times 5^0$

is of the form $2^m \times 5^n$, where m = 3 and n = 0.

So, the given number has terminating decimal expansion.

$\therefore \frac{3\times5^3}{2^3\times5^3} = \frac{3\times125}{(2\times5)^3} = \frac{375}{(10)^3} = \frac{375}{1000} = 0.375$

26. Suppose α,β and γ are the zeros of the said polynomial p(x)

Then, we have α = 2, β = -3 and γ = 4

Now,

$\alpha+\beta+\gamma$=2-3+4=3......(1)

$\alpha\beta+\beta\gamma+\gamma\alpha$=2(-3)+(-3)(4)+(4)(2)=-6-12+8=-10......(.2)

$\alpha\beta\gamma$=2(-3)(4)=-24 ....(3)

Now, a cubic polynomial whose zeros are $\alpha,\ \beta\ and\ \gamma$ is given by

p(x) = x^3 - $(\alpha+\beta+\gamma)x^2$ + $(\alpha\beta+\beta\gamma+\gamma\alpha)$x - $\alpha\beta\gamma$

Now putting the values from (1),(2) and (3) we get

p(x) = x^3 -(3)x^2 + (-10)x - (-24)

= x^3 -3x^2 -10x +24

OR

The given polynomial p(x)=3x^3+16x^2+15x-18

Since $\frac{2}{3}$ is a zero of p(x), so $\left(x - \frac{2}{3}\right)$ is a factor of p(x).

⇒ (3x - 2) is also factor of p(x)

On dividing p(x) by (3x - 2) , we get

```
              x² + 6x + 9
3x – 2)  3x³ + 16x² + 15x – 18
      (–) 3x³ – 2x²
           –     +
      ----------------------
             18x² + 15x – 18
          (–)18x² – 12x
              –      +
          ------------------
                  27x – 18
              (–) 27x – 18
                   –     +
              -------------
                     0
```

So f(x)=(3x-2)(x^2+6x+9)

= (x^2 + 3x + 3x + 9)(3x - 2)

=[x(x+3)+3(x+3)](3x-2)

= (x + 3)(x + 3)(3x - 2)

$\therefore$ x = -3 or x = -3 or x = $\frac{2}{3}$

Thus, the other two zeros are -3, -3.

27. Assume side of one square = x m and side of other square = y m, then we have

 9x = 4y + 1

 $\Rightarrow \frac{9x-1}{4}$ = y ....(i)

 According to given situation we have,

 $6y^2 = 29x^2 + 1$

 $\Rightarrow 6(\frac{9x-1}{4})^2 = 29x^2 + 1$

 $\Rightarrow \frac{3(81x^2-18x+1)}{8} = 29x^2 + 1$

 $\Rightarrow 243x^2 - 54x + 3 = 232x^2 + 8$

 $\Rightarrow 11x^2 - 54x - 5 = 0$

 Factorize above quadratic equation we get

 $\Rightarrow$(x - 5)(11x + 1) = 0

 $\Rightarrow$x = 5 or x = $\frac{-1}{11}$ (negative value is rejected)

 $\therefore$x = 5m

 When x = 5, then y = $\frac{9\times5-1}{4}$ = 11m (From (i))

 Hence sides of the square are 5m and 11m.

28. Given,

 $x = \frac{1}{2-\frac{1}{2-\frac{1}{2-x}}}$

 Now, $2 - \frac{1}{2-x} \Rightarrow \frac{2(2-x)-1}{(2-x)} \Rightarrow \frac{4-2x-1}{2-x} \Rightarrow \frac{3-2x}{2-x}$

 and $2 - \frac{1}{2-\frac{1}{2-x}} \Rightarrow 2 - \frac{2-x}{3-2x} \Rightarrow \frac{2(3-2x)-(2-x)}{3-2x} \Rightarrow \frac{4-3x}{3-2x}$

 $Hence,\ x = \frac{3-2x}{4-3x}$

 Cross multiplication,

 $\Rightarrow$ x(4 - 3x) = (3 - 2x)

 $\Rightarrow 4x - 3x^2 = 3 - 2x$

 $\Rightarrow 3x^2 - 6x + 3 = 0 \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow (x - 1)^2 = 0 \Rightarrow x = 1, 1$

29. The given equations are

 $6x + 5y = 7x + 3y + 1 = 2(x + 6y - 1)$

 Therefore,we have

 $6x + 5y = 2(x + 6y - 1)$

 $6x + 5y = 2x + 12y - 2$

 $6x - 2x + 5y - 12y = -2$

 $4x - 7y = -2$......(i)

 Also,

 $7x + 3y + 1 = 2(x + 6y - 1)$

 $7x + 3y + 1 = 2x + 12y - 2$

 $7x - 2x + 3y - 12y = -2 - 1$

 $5x - 9y = -3$.........(ii)

 Multiplying (i) by 9 and (ii) by 7, we get

 $36x - 63y = -18$ ......(iii)

 $35x - 63y = -21$ .....(iv)

 Subtracting (iii) and (iv),we get

 $x = 3$

 Substituting x = 3 in (i),we get

 $\Rightarrow 4 \times 3 - 7y = -2$

 $\Rightarrow -7y = -2 - 12$

 $\Rightarrow -7y = -14$

$\Rightarrow$y = 2

$\therefore$ Solution is x = 3, y = 2

OR

Let the number of pens be x and that of pencil be y,

As per given condition

Reena has total 40 pens and pencils.

So, x + y =40 ....(i)

And if she has 5 more pencils and 5 less pens, then number of pencils would become 4 times the number of pens.

So, y + 5 = 4(x- 5)

y + 5 = 4x - 20

4x - y = 25 .... (ii)

Adding equation (i) and equation (ii), we get

x + 4x = 40 + 25

5x = 65

$\Rightarrow x = \frac{65}{5} = 13$

Putting x = 13 in equation (i), we get

13 + y = 40

y = 40 - 13 = 27

Hence, Reena has 13 pens and 27 pencils.

30. Let take that 3 + 2$\sqrt{5}$ is a rational number.

So we can write this number as

3 + 2$\sqrt{5}$ = $\frac{a}{b}$

Here a and b are two co-prime numbers and b is not equal to 0.

Subtract 3 both sides we get,

2$\sqrt{5}$ = $\frac{a}{b}$ - 3

2$\sqrt{5}$ = $\frac{a-3b}{b}$

Now divide by 2 we get

$\sqrt{5}$ = $\frac{a-3b}{2b}$

Here a and b are an integer so $\frac{a-3b}{2b}$ is a rational number so $\sqrt{5}$ should be a rational number but $\sqrt{5}$ is an irrational number so it contradicts the fact.

Hence the result is 3 + 2$\sqrt{5}$ is an irrational number

Now its square will again contain an irrational number.

Hence the given number is an irrational number.

31. We know that, if x = a is a zero of a polynomial then x - a is a factor of quadratic polynomials.

Since $\frac{-1}{4}$ and 1 are zeros of polynomial.

Therefore $\left(x+\frac{1}{4}\right)$ (x - 1)

$= x^2 + \frac{1}{4}x - x - \frac{1}{4}$

$= x^2 + \frac{1}{4}x - \frac{4}{4}x - \frac{1}{4}$

$= x^2 + \frac{1-4}{4}x - \frac{1}{4}$

$= x^2 - \frac{3}{4}x - \frac{1}{4}$

Hence, the family of quadratic polynomials is f(x) = $k\left(x^2 - \frac{3}{4}x - \frac{1}{4}\right)$, where k is any non-zero real number.

32. Assume f(x) = $6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$

If $\sqrt{2}$ is the zero of f(x), then $\left(x-\sqrt{2}\right)$ will be a factor of f(x). So, by remainder theorem when f(x) is divided by $\left(x-\sqrt{2}\right)$, the quotient comes out to be quadratic.

Now we divide $6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$ by $\left(x-\sqrt{2}\right)$.

$$\begin{array}{r} 6x^2+7\sqrt{2}x+4 \\ x-\sqrt{2}\,\big)\overline{6x^3+\sqrt{2}x^2-10x-4\sqrt{2}} \\ \underline{-6x^3\mp 6\sqrt{2}x^2} \\ 7\sqrt{2}x^2-10x-4\sqrt{2} \\ \underline{-7\sqrt{2}x^2\mp 14x} \\ 4x-4\sqrt{2} \\ \underline{-4x\mp 4\sqrt{2}} \\ 0 \end{array}$$

$\therefore\ f(x)=(x-\sqrt{2})(6x^2+7\sqrt{2}x+4)$ (By Euclid's division algorithm)
$=(x-\sqrt{2})(6x^2+4\sqrt{2}x+3\sqrt{2}x+4)$ (By factorization method)
For zeroes of f(x), put f(x) = 0
$\therefore (x-\sqrt{2})(6x^2+4\sqrt{2}x+3\sqrt{2}x+4)=0$
$\Rightarrow\ (x-\sqrt{2})[2x(3x+2\sqrt{2})+\sqrt{2}(3x+2\sqrt{2})]=0$
$\Rightarrow (x-\sqrt{2})(3x+2\sqrt{2})(2x+\sqrt{2})=0$
$\Rightarrow x-\sqrt{2}=0$ or $3x+2\sqrt{2}=0$ or $2x+\sqrt{2}=0$
$\Rightarrow x=\sqrt{2}$ or $x=\frac{-2\sqrt{2}}{3}$ or $x=\frac{-\sqrt{2}}{2}$
So, other two roots are $=\frac{-2\sqrt{2}}{3}$ and $\frac{-\sqrt{2}}{2}$.

33. Here, x - 3 is the HCF of
x^3-2x^2+px+6 and x^2-5x+q
SInce x - 3 is a common factor of given expression
$f(x)=x^3-2x^2+px+6$,then by factor theorem
f(x) = 0
$\Rightarrow 3^3-2\times 3^2+\mathrm{p}\times 3+6=0$
$\Rightarrow 27-18+3p+6=0\Rightarrow 15+3p=0$
$\Rightarrow 3p=-15\Rightarrow p=\frac{-15}{3}=-5$
SInce x -3 is a factor of g(x) = x^2-5x+q,
then by factor theorem, g(3) = 0
$\Rightarrow 3^2-5\times 3+q=0\Rightarrow 9-15+q=0$
$\Rightarrow -6+q=0\Rightarrow q=6$
$\therefore 6p+5q=6\times(-5)+5\times 6$
$=-30+30=0$
Hence 6p + 5q = 0

OR

Let a be the positive integer and b = 6.
Then, by Euclid's algorithm, a = 6q + r for some integer q ≥ 0 and r = 0, 1, 2, 3, 4, 5 because 0 ≤ r < 5.
So, a = 6q or 6q + 1 or 6q + 2 or 6q + 3 or 6q + 4 or 6q + 5.
$(6q)^2 = 36q^2 = 6(6q^2)$
= 6m, where m is any integer.
$(6q + 1)^2 = 36q^2 + 12q + 1$
$= 6(6q^2 + 2q) + 1$
= 6m + 1, where m is any integer.
$(6q + 2)^2 = 36q^2 + 24q + 4$
$= 6(6q^2 + 4q) + 4$
= 6m + 4, where m is any integer.
$(6q + 3)^2 = 36q^2 + 36q + 9$
$= 6(6q^2 + 6q + 1) + 3$
= 6m + 3, where m is any integer.
$(6q + 4)^2 = 36q^2 + 48q + 16$
$= 6(6q^2 + 7q + 2) + 4$

= 6m + 4, where m is any integer.

$(6q + 5)^2 = 36q^2 + 60q + 25$

$= 6(6q^2 + 10q + 4) + 1$

= 6m + 1, where m is any integer.

Hence, The square of any positive integer is of the form 6m, 6m + 1, 6m + 3, 6m + 4 and cannot be of the form 6m + 2 or 6m + 5 for any integer m.

34. It is given that on dividing the polynomial $4x^4 - 5x^3 - 39x^2 - 46x - 2$ by the polynomial g(x), the quotient is $x^2 - 3x - 5$ and the remainder is $- 5x + 8$. We have to find the polynomial g(x).

Now, we know that

Dividend = (Divisor × Quotient) + Remainder

$4x^4 - 5x^3 - 39x^2 - 46x - 2 = g(x)\,(x^2 - 3x - 5) + (-5x + 8)$

or, $4x^4 - 5x^3 - 39x^2 - 46x - 2 + 5x - 8 = g(x)\,(x^2 - 3x - 5)$

or, $4x^4 - 5x^3 - 39x^2 - 41x - 10 = g(x)(x^2 - 3x - 5)$

$g(x) = \frac{4x^4 - 5x^3 - 39x^2 - 41x - 10}{(x^2 - 3x - 5)}$

$$\begin{array}{r|l} & 4x^2 + 7x + 2 \\ x^2 - 3x - 5 & 4x^4 - 5x^3 - 39x^2 - 41x - 10 \\ & 4x^4 - 12x^3 - 20x^2 \\ & - \quad + \quad + \\ \hline & 7x^3 - 19x^2 - 41x - 10 \\ & 7x^3 - 21x^2 - 35x \\ & - \quad + \quad + \\ \hline & 2x^2 - 6x - 10 \\ & 2x^2 - 6x - 10 \\ & - \quad + \quad + \\ \hline & 0 \end{array}$$

Hence, $g(x) = 4x^2 + 7x + 2$

35. We can rewrite the equations as:

x - y = - 3 and 2x + 3y = 4

For equation, x - y = - 3

First, take x = 0 and find the value of y.

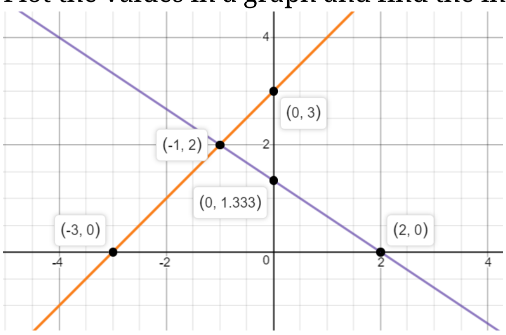
Then, take y = 0 and find the value of x.

x	0	-3
y	3	0

Now similarly solve for equation, 2x + 3y = 4

x	0	2
y	$\frac{4}{3}$	0

Plot the values in a graph and find the intersecting point for the solution.



Hence, the solution so obtained from the graph is (-1, 2), which is the intersecting point of the two lines.

Clearly, from the graph we can identify base and height of the triangle.

Now, we know

Area of Triangle = $\frac{1}{2} \times$ base $\times$ height

Thus, Area($\triangle$ABC) = $\frac{1}{2} \times 5 \times 2$

[∵ Base = BO + OC = 3 + 2 = 5 units and height = 2 units]

Area($\triangle$ABC) = 5 sq. units

OR

Let the time taken by the smaller pipe to fill the tank be x hr.

Time taken by the larger pipe = (x - 10) hr

Part of the tank filled by a smaller pipe in 1 hour $= \frac{1}{x}$

Part of the tank filled by the larger pipe in 1 hour $= \frac{1}{x-10}$

It is given that the tank can be filled in $9\frac{3}{8} = \frac{75}{8}$ hours by both the pipes together. So $\frac{75}{8}$ hours, multiplied by the sum of parts filled with both pipes in one hour equal to complete work i.e 1.

$\frac{75}{8}\left(\frac{1}{x} + \frac{1}{x-10}\right) = 1$

$\Rightarrow \frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$

$\Rightarrow \frac{x-10+x}{x(x-10)} = \frac{8}{75}$

$\Rightarrow \frac{2x-10}{x(x-10)} = \frac{8}{75}$

$\Rightarrow 75(2x - 10) = 8x^2 - 80x$

$\Rightarrow 150x - 750 = 8x^2 - 80x$

$\Rightarrow 8x^2 - 230x + 750 = 0$

Now for factorizing the above quadratic equation, two numbers are to be found such that their product is equal to 750×8 and their sum is equal to 230

$\Rightarrow 8x^2 - 200x - 30x + 750 = 0$

$\Rightarrow 8x(x - 25) - 30(x - 25) = 0$

$\Rightarrow (x - 25)(8x - 30) = 0$

$\Rightarrow x = 25, \frac{30}{8}$

Time taken by the smaller pipe cannot be $\frac{30}{8} = 3.75$ hours.

As in this case, the time taken by the larger pipe will be negative, which is logically not possible.

Therefore, time taken individually by the smaller pipe and the larger pipe will be 25 and 25 - 10 = 15 hours respectively.

36. Factors of $x^2 + 7x + 12$:

$x^2 + 7x + 12 = 0$

$\Rightarrow x^2 + 4x + 3x + 12 = 0$

$\Rightarrow x(x + 4) + 3(x + 4) = 0$

$\Rightarrow (x + 4)(x + 3) = 0$

$\Rightarrow x = -4, -3$ ...(i)

Since $p(x) = x^4 + 7x^3 + 7x^2 + px + q$

If p(x) is exactly divisible by $x^2 + 7x + 12$, then x = - 4 and x = - 3 are its zeroes. So putting x = - 4 and x = - 3.

$p(-4) = (-4)^4 + 7(-4)^3 + 7(-4)^2 + p(-4) + q$

but p(- 4) = 0

∴ 0 = 256 - 448 + 112 - 4p + q

0 = - 4p + q - 80

$\Rightarrow$4p - q = - 80 ...(i)

and $p(-3) = (-3)^4 + 7(-3)^3 + 7(-3)^2 + p(-3) + q$

but p(-3) = 0

$\Rightarrow$0 = 81-189 + 63 - 3p + q

$\Rightarrow$0 = -3p + q -45

$\Rightarrow$3p -q = -45 ....(ii)

$$\begin{array}{r} 4p - q = -80 \\ 3p - q = -45 \\ - \quad + \quad\quad + \\ \hline p = -35 \end{array}$$

On putting the value of p in eq. (i),we get,

$4(-35) - q = -80$

$\Rightarrow -140 - q = -80$

$\Rightarrow\ -q = 140 - 80$

$\Rightarrow\ \ -q = 60$

$\therefore q = -60$

Hence, p = -35, q = -60